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Gravitational Instability of Gas with Thermal Effects

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Abstract- *The effect of self-gravity on the thermal waves in a non – reacting arbitrary fluid including thermal conductivity and heat loss function. We are studing the stability of a fluid by imposing perturbations* $\sim$ *exp* $(\omega t + ik.r)$ *on the equlibrium state and neglecting quadratic and higher powers in the gas dynamic equations.The resulting secular equation is a polynomial with a power equal to the number of independent modes of the system. Instability and the stability conditions in various cases of interest have been stablished.*

Key Words: *Gravitational Instability , perturbation , thermal conductivity , equilibrium state ,Heat loss function.*

I. Introduction

In recent years large number of workers have devoted their efforts to the understanding of gravitational instabilities and problems of star formation. Silk [1] has shown the importance of thermal instability in uniformly collapsing primordial clouds. Chieze [2] has suggested that in the early stage of the formation of molecular clouds fragmentation occurs near the gravitational instability threshold. Reddish and Wickram singhe [3] have also shown that hydrogen is in solid state in the inter-stellar clouds where star formation is just setting in and thought that an instability associated with condensation of solid H_2 leads to fragmentation of clouds. Alfven and Carlqvist [4] have suggested that stars are formed out of dusty cloud plasma in which particles of large variety and mass exist. Boss [5] has investigated the problem of inner core formation in the process of star formation. Thermal conductivity is also an important parameter affecting the formation of a star from dusty-gas or dusty plasma. Field [6] has studied thermal instability of uniform static, optically thin medium which occurs due to density and temperature dependence of heat loss function ζ.

In the light of above we have studied the effect of self-gravity on the thermal waves in anon reacting arbitrary fluid including thermal conductivity and heat loss function. We are studying the stability of a fluid by imposing perturbations on the equilibrium state and neglecting quadratic and higher powers in the gas dynamic equations. The resulting instability and stability conditions in various cases of interest have been stablished.

II. Linearized Perturbation Equations

The linearized perturbation equations of such amedium in the presence of self-gravity are-

$$\omega\rho_1+ik\rho_0 v_1=0, \qquad ....(1)$$

$$Ikp_1+\rho_0\omega v_1-\frac{\rho_0 4\pi G \rho_1 ik}{k^2}=0 , \qquad ....(2)$$

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$$\frac{R\omega}{\mu(\gamma-1)} T_1 - \frac{R}{\mu}\frac{T_0}{\rho_0}\omega\rho_1 + \zeta_\rho \rho_1 \ \zeta_T T_1 + \frac{1}{\rho}\ \theta k^2 T_1 = 0, \qquad ....(3)$$

$$\frac{p_1}{p_0} - \frac{T_1}{T_0} - \frac{\rho_1}{\rho_0} = 0, \qquad ....(4)$$

Where ρ , v , p , T , G and θ are mass density, gas velocity, pressure, temperature, gravitational potential and coefficient of thermal conductivity respectively. ζ (ρ,T) is the heat loss functiondefined as the net cooling rate per unit mass and time, and R is the gas constant. In the above equations the suffix 0 refers to variables in the equilibrium state , which are assumed to be independent of space and time. We omit this for simplicity in further discussion.

III. Dispersion Relation

We can make determinant of the matrix with the help of equations (1-4) and after solving that gives the dispersion relation,

$$\omega^3 + \omega^2 \left[\frac{\mu(\gamma-1)}{R}\left\{\zeta_T + \frac{\theta k^2}{\rho}\right\}\right] + \omega\Omega_j^2 + (\gamma-1)\frac{\mu}{R}\,\Omega_I^2\,\frac{\theta k}{\rho} = 0 \qquad ....(5)$$

Where,

$$C^2 = \frac{\gamma p}{\rho}, \quad c'^2 = \frac{p}{\rho}, \quad \Omega_j^2 = c^2k^2 - 4\pi G\rho, \quad \Omega_I^2 = c'^2k^2 - 4\pi G\rho \qquad ....(6)$$

Here c and c' are the adiabatic and isothermal velocities of sound. And $\theta = \frac{K_T}{\rho C_p}$ is the thermometric conductivity.

Equation (5) shows the dispersion relation for an ideal gas with thermal conductivity and heat loss function for infinite homogeneous self-gravitating fluid.

IV. Discussion

If we neglect the effect of heat loss function the equation (5) represents an infinite homogeneous , self-gravitating ideal gas , and becomes

$$\omega^3 + \omega^2 \left\{\frac{\mu(\gamma-1)}{R}\frac{\theta k^2}{\rho}\right\} + \omega\Omega_j^2 + (\gamma-1)\frac{\mu}{R} = 0 \qquad ....(7)$$

Hence it follows that, when

$$\Omega_I^2 = c'^2k^2 - 4\pi G\rho < 0,$$

One of the roots of equation (7) is positive , meaning thereby instability, with the condition

$$K^2 < k_{j_1}^2 = \frac{4\pi G\rho}{c'^2}, \qquad ....(8)$$

If thermal conductivity is not considered i.e. θ = 0 in equation (5), will become as

$$\omega^3 + \omega^2 \left\{\frac{\mu(\gamma-1)}{R}\zeta_T\right\} + \omega\Omega_j^2 + \left[(\gamma-1)\left\{\frac{\mu}{R}\Omega_I^2\zeta_T - k^2\rho\,\zeta_\rho\right\}\right] = 0, \qquad ....(9)$$

From the constant term of dispersion relation (9) we have

$$\frac{\mu}{R}\ \Omega_I^2\ \zeta_T - k^2\ \rho\,\zeta_\rho < 0, \qquad ....(10)$$

or

$$k^2 < k_{j_2}^2 = \frac{4\pi G\rho}{c'^2 - \frac{\zeta_\rho}{\zeta_T}\ \rho}$$

The system is unstable for all wave numbers $k < k_{j_2}$ and k_{j_2} is modified Jeans wave number.

If we neglect the effect of both heat loss function and thermal conductivity, the dispersion relation (5) will be as follows,

$$\omega^2+\Omega_j^2=0 \qquad ....(11)$$

From eq. (11) we find the condition as follows

When $\Omega_j^2 < 0$ ....(12)

or

$$k^2 < k_{j_s}^2 = \frac{4\pi G\rho}{c^2}, \qquad ..(13)$$

where k_{j_s} is the Jeans criterion.

The system is unstable for all wave numbers $k < k_{j_s}$.

Now we discuss the condition of instability for general dispersion relation obtained as follows. From the constant term of dispersion relation (5) we get the condition of instability as

$$k^2 < k_{j_4}^2 = \frac{1}{2}\Bigg[-\frac{\rho}{\theta}\zeta_T + \frac{4\pi G\rho}{C'^2} + \frac{\rho^2\zeta_\rho R}{c'^2\mu\theta} \pm \sqrt{\left(\frac{\rho}{\theta}\zeta_T\right)^2 + \left(\frac{4\pi G\rho}{c'^2}\right)^2 + \left(\frac{\rho^2\zeta_\rho R}{C'^2\mu\theta}\right)^2 + 2\frac{\rho}{\theta}\zeta_T\frac{4\pi G\rho}{C'^2} + \cdot\ \rho^2\frac{b}{z} + 2\frac{4\pi G\rho}{c'^2}\frac{\rho^2\zeta_\rho R}{c'^2\mu\theta}}\Bigg] .$$

....(14)

which depends on gravity, thermal conductivity and heat loss function.

V. Conclusion

We see that for an infinite homogeneous self-gravitating fluid of finite thermal conductivity the Jeans condition is satisfied, then the system is unstable. 0n comparing equations (8) and (13), we may conclude that the adiabatic speed of sound is being replaced by the isothermal one. When we compare the equations (10) and (13), we can say that heat loss function also changes the velocity of sound, and modifies the condition of instability by increasing the Jeans wave number.

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