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The Effect of Surface Tension along with FLR Correction and Suspended Particle on the Kelvin Helmholtz Instability of Viscous Fluids

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Abstract *- The Kelvin Helmholtz instability for an interface separating two viscous and incompressible hydromagnetic fluids is investigated in the presence of surface tension along with suspended particles and FLR (finite Larmor radius) correction. The dispersion relation for such a medium is obtained by using normal mode technique. A dispersion relation is obtained by solving the linearized equations of the considered system with appropriate boundary conditions. We have calculated the growth rate of Kelvin Helmholtz instability and found that the suspended particles has destabilizing effect and surface tension has the stabilizing effect.*

Keywords: *Finite Larmor radius, K-H instability, normal mode method, suspended particles*

I. Introduction

The Kelvin Helmholtz instability in magneto hydrodynamic fluids arises at the interface of two superposed fluids due to the velocity shear. Kelvin-Helmholtz instability has importance in magnetosphere and phenomenon's of space and astrophysics e.g. mixing of clouds, entering the meteor on the earth's atmosphere. In this perspective, the KH instability has been discussed by many authors. Chandrasekhar [1], Michael [2], Gerwin [3] have discussed Kelvin-Helmholtz instability of incompressible fluids with different parameters. Sanghvi and Chhajlani [4] have investigated the combined effect of finite Larmor radius correction and suspended particles on Kelvin Helmholtz instability of incompressible medium in the presence of uniform magnetic field. El-Sayed [5] has analysed the combined effect of FLR, suspended particle and viscosity on Kelvin Helmholtz instability of two superposed incompressible fluids in the presence of uniform magnetic field. El-Sayed [6] examined the hydromagnetic instability of fluid particle flow in oldroydian viscoelastic porous media and discussed the effects of suspended dust particles by taking uniform magnetic field.

Sharma and Chhajlani [7] discussed the RT instability of two superposed plasmas, consisting of interacting ions and neutrals, in a horizontal magnetic field and found that RT instability remains unaffected by permeability of the porous medium along with neutral particles and rotation. Recently Prajapati and Chhajlani [8] have studied KH instability of magnetized plasma with surface tension and dust particles.

Prajapati and Chhajlani [9] have discussed the effect of surface tension and suspended particles on the K-H instability. Kumar and Mohan [10] have investigated the Kelvin Helmholtz instability of two viscoelastic fluids.

Keeping in mind the various applications mentioned above our interest in the present paper, is to study the combined effect of finite Larmor radius and surface tension on the Kelvin Helmholtz instability of highly viscous fluids in presence of suspended dust particle.

II. Formulation of problem

We consider two viscous, incompressible, homogeneous isotropic fluids are separated by a plane interface (z=0). Fluid in the region $z < 0$ is denoted by subscript 1 and the fluid in the region $z > 0$ denoted by subscript 2. The medium is assumed to be uniform mixture of conducting fluid and non-conducting suspended dust particles, The fluid with suspended particles is streaming with velocity **U** (0, U, 0) in action of transverse uniform magnetic field **B** (H, 0, 0). We assuming uniform sized (of radius a) and spherical shaped suspended particles, exerts a force KN (V-U) per unit volume on the fluid, where V and N represents the velocity and number density of suspended particles. The constant K=6πaμ, denotes Stokes drag coefficient.

Let **Π**, μ, p, μ_e and ρ represent the stress tensor, viscosity, pressure, permeability of magnetic field and density respectively. Then the relevant linearized perturbed equations of the problem are

$$\rho\left[\frac{\partial}{\partial t}+(\mathbf{U}.\nabla)\right]\mathbf{u} = -\nabla\ \delta p - \nabla\cdot\mathbf{\Pi} + KN\ (\mathbf{v}-\mathbf{u}) + \frac{\mu_e}{4\pi}\left[(\nabla\times\mathbf{B})\times\boldsymbol{\delta}\mathbf{B} + (\nabla\times\boldsymbol{\delta}\mathbf{B})\times\mathbf{B}\right] + \mu\nabla^2\mathbf{u} + \left(\frac{\partial w}{\partial \mathbf{X}}+\frac{\partial \mathbf{u}}{\partial z}\right)\frac{\partial \mu}{\partial z} + \Sigma_s\left[T_s\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\delta z_s\right]\delta(z-z_s) \quad \textbf{(1)}$$

$$\left(\frac{\partial}{\partial t}+(\mathbf{U}\cdot\nabla)\right)\delta\rho + (\mathbf{u}\cdot\nabla)\rho = 0 \quad (2)$$

$$\nabla\cdot\mathbf{u} = 0 \quad (3)$$

$$\nabla\cdot\boldsymbol{\delta}\mathbf{B} = 0 \quad (4)$$

$$\left[\tau\left(\frac{\partial}{\partial t}+(\mathbf{U}\cdot\nabla)\right)+1\right]\mathbf{v} = \mathbf{u} \quad (5)$$

$$\frac{\partial\boldsymbol{\delta}\mathbf{B}}{\partial t} + (\mathbf{U}.\nabla)\boldsymbol{\delta}\mathbf{B} = (\mathbf{B}.\nabla)u \quad (6)$$

$$\left[(\mathbf{U}.\nabla)+\frac{\partial}{\partial t}\right]\delta z = w \quad (7)$$

Here δp, δρ **u** (u,v,w), v, **δB**(δB_x, δB_y, δB_z), represent the perturbation in pressure, density, fluid velocity, velocity of suspended particles and magnetic field of the medium respectively . Here τ = m/K is relaxation time for the suspended particle and mN is the mass of the particles per unit volume.

If $\nu_0 = \frac{R_L^2\Omega_L}{4}$ where R_L is Finite Larmor Radius and Ω_L is ion gyro frequency then the Components of stress tensor **Π** in horizontal magnetic field **B** (B, 0, 0) are,

$$\Pi_{xx} = 0$$

$$\Pi_{yy} = -\Pi_{zz} = -\rho\nu_0\left(\frac{\partial w}{\partial y}+\frac{\partial v}{\partial z}\right)$$

$$\Pi_{xy} = \Pi_{yx} = -2\rho\nu_0\left(\frac{\partial u}{\partial z}\right)$$

$$\Pi_{xz} = \Pi_{zx} = 2\rho\nu_0\left(\frac{\partial u}{\partial y}\right)$$

$$\Pi_{zy} = \Pi_{yz} = \rho\nu_0\left(\frac{\partial v}{\partial y}-\frac{\partial w}{\partial z}\right) \quad (8)$$

To solve the linearized equations we take space and time dependent perturbation,

$$\exp(iky + nt) \quad (9)$$

Here k is wave number and n is growth rate of perturbation.

After solving and eliminating some variables from the above equations (1) to (9), we get an equation in z component of velocity as follows.

$$\sigma\left[1+\frac{\alpha}{\tau\sigma+1}\right]\left[D(\rho Dw) - k^2\rho w\right] + 2i\nu_0 k\left[D(D\rho\, Dw) - k^2(D\rho)w\right] - \left[\begin{array}{l} D\{\mu(D^2-k^2)Dw\} \\ -\mu k^2(D^2-k^2)w \end{array}\right] - \left[\begin{array}{l} D\{(D^2+k^2)w(D\mu)\} \\ -2k^2(D\mu)(Dw) \end{array}\right] - \frac{\Sigma T_s k^4 w\,\delta(z-z_s)}{\sigma} = 0 \quad (10)$$

Here $\sigma = n + ikU$,

$\alpha = mn/\rho$ and $D = d/dz$

The above equation (10) shows the effect of FLR, suspended particles and surface tension on the streaming viscous superposed fluids. If we ignore the effect of surface tension in the above equation, reduced to similar equation obtained by El-Sayed [5] .

III. Dispersion relation for superposed fluids

The dispersion relation for two superposed fluids occupying the region z>0 and z<0, are separated by horizontal boundary z=0 of constant density and viscosity, equation (10) reduced into the following form

$$\left(D^2-k^2\right)\left(D^2-q_j^2\right)w_j=0 \qquad (11)$$

Where $q_j^2 = k^2 + \frac{\sigma_j}{\upsilon_j}\left[1+\frac{\alpha_j}{\tau\sigma_j+1}\right]$

$j = 1, 2 \qquad (12)$

The general solution of equation (11) as given below

$$w_j = \left[A_j \exp\left(\pm kz\right) + B_j \exp\left(\pm q_j z\right)\right]\sigma_j \qquad (13)$$

Where A_j and B_j, are arbitrary constants, to be determined by using Chandrasekhar's [1] boundary conditions of the problem we get the following equation.

$$\begin{aligned}
&-2k\begin{bmatrix} k\left(q_1-k\right)\left(\beta_1\upsilon_1-\beta_2\upsilon_2\right)- \\ \beta_1\sigma_1\left\{1+\frac{\alpha_1}{\tau\sigma_1+1}\right\}\end{bmatrix}
\begin{bmatrix} \beta_2\sigma_2^2\left\{1+\frac{\alpha_2}{\tau\sigma_2+1}\right\}+ \\ k\left(\beta_1\upsilon_1\sigma_1-\beta_2\upsilon_2\sigma_2\right)\left(q_2-k\right)\end{bmatrix} \\
&+2k\begin{bmatrix} k\left(q_2-k\right)\left(\beta_1\upsilon_1-\beta_2\upsilon_2\right)+ \\ \beta_2\sigma_2\left\{1+\frac{\alpha_2}{\tau\sigma_2+1}\right\}\end{bmatrix}
\begin{bmatrix} \beta_1\sigma_1^2\left\{1+\frac{\alpha_1}{\tau\sigma_1+1}\right\}- \\ k\left(\beta_1\sigma_1\upsilon_1-\beta_2\sigma_2\upsilon_2\right)\left(q_1-k\right)\end{bmatrix} \\
&+\begin{bmatrix} \left(q_1-k\right)\beta_2\sigma_2\left\{1+\frac{\alpha_2}{\tau\sigma_2+1}\right\}+ \\ \left(q_2-k\right)\beta_1\sigma_1\left\{1+\frac{\alpha_1}{\tau\sigma_1+1}\right\}\end{bmatrix} \qquad (14) \\
&\begin{bmatrix} 2i\nu_0k^2\left(\beta_2\sigma_2-\beta_1\sigma_1\right)+ \\ \beta_1\sigma_1^2\left\{1+\frac{\alpha_1}{\tau\sigma_1+1}\right\} \\ +\beta_2\sigma_2^2\left\{1+\frac{\alpha_2}{\tau\sigma_2+1}\right\}+\frac{Tk^3}{\rho_1+\rho_2}\end{bmatrix}=0
\end{aligned}$$

For two highly viscous fluids from equation (12) we get,

$$q_j-k=\frac{\sigma}{2\upsilon_j k}\left[1+\frac{\alpha_j}{\tau\sigma_j+1}\right] \qquad (15)$$

Now by substituting the value of $\left(q_1-k\right)$ and $\left(q_2-k\right)$ from the equation (15), in the equation (14) we obtain the general dispersion relation

$$\begin{aligned}
&2k^2\left(\beta_1\sigma_1\upsilon_1+\beta_2\sigma_2\upsilon_2\right)+ \\
&2i\nu_0k^2\left(\beta_2\sigma_2-\beta_1\sigma_1\right)+\frac{Tk^3}{\rho_1+\rho_2}+ \\
&\beta_1\sigma_1^2\left\{1+\frac{\alpha_1}{\tau\sigma_1+1}\right\}+ \qquad (16) \\
&\beta_2\sigma_2^2\left\{1+\frac{\alpha_2}{\tau\sigma_2+1}\right\}=0
\end{aligned}$$

Here $\sigma_1 = n+ikU_1$, $\sigma_2 = n+ikU_2$

$$\alpha_1=\frac{mN}{\rho_1},\ \alpha_2=\frac{mN}{\rho_2}$$

$$\beta_1=\frac{\rho_1}{\rho_1+\rho_2},\ \text{and}\ \beta_2=\frac{\rho_1}{\rho_1+\rho_2}$$

IV. Discussion

Consider now the special case by using following assumption

$\alpha_1=\alpha_2=\alpha_0, \beta_1=\beta_2=1/2,$

$U_1=U, U_2=-U, \upsilon_1=\upsilon_2=\upsilon, f_s=1/\tau$

$\sigma_2=n-ikU$, $\sigma_1=n+ikU$

dispersion relation (16) reduces to

$$\begin{aligned}
&n^4+n^3\left[2k^2\upsilon+f_s\left(2+\alpha_0\right)\right]+ \\
&n^2\begin{bmatrix}\left(1+\alpha_0\right)f_s^2+4f_sk^2\upsilon \\ +2kU\nu_0k^2+\frac{Tk^3}{2\rho}\end{bmatrix}+ \\
&n\begin{bmatrix}2k^2f_s^2\upsilon+f_sk^2U^2\left(\alpha_0-2\right)+ \\ 4f_skU\nu_0k^2+f_s\frac{Tk^3}{\rho}+2k^4U^2\upsilon\end{bmatrix}+ \\
&\begin{bmatrix}\left\{2kU\nu_0k^2-k^2U^2\right\}\left\{f_s^2+k^2U^2\right\}- \\ f_s^2k^2U^2\alpha_0+\frac{Tk^3}{2\rho}\left(f_s^2+k^2U^2\right)\end{bmatrix}=0 \qquad (17)
\end{aligned}$$

Here fs = relaxation frequency of the suspended particles.

Dispersion relation (17) represents the effect of suspended particle, FLR, surface tension and viscosity on the KH instability. If we ignore the effect of Surface-tension in the dispersion relation (17), we found the similar equation (69) obtained by El-Sayed [5].

To perform numerical calculation, we write the dimensionless form of the dispersion relation (17) by using dimensionless parameters given below

$$n^* = \frac{nL}{V_A},\ k^* = kL,\ U^* = \frac{U}{V_A},\ \nu_0^* = \frac{\nu_0}{V_A L},$$

$$\upsilon^* = \frac{\upsilon}{V_A L},\ T^* = \frac{T}{V_A^2},\ \rho^* = \rho L$$

Here V_A is Alfven velocity and L is characteristic length. By using these parameters equation (17) transformed to

$$n^{*4} + n^{*3}\left[2k^{*2}\upsilon^* + f_s^*(2+\alpha_0)\right] + n^{*2}\left[\begin{array}{l}(1+\alpha_0)f_s^{*2} + 4f_s^* k^{*2}\upsilon^* + \\ 2k^{*3}U^*\nu_0^* + \dfrac{T^* k^{*3}}{2\rho^*}\end{array}\right] + n^*\left[\begin{array}{l}2k^{*2} f_s^{*2}\upsilon^* + f_s^* k^{*2} U^{*2}\alpha_0 - \\ 2f_s^* k^{*2} U^{*2} + \\ 4f_s^* k^{*3} U^*\nu_0^* + \\ f_s^*\dfrac{T^* k^{*3}}{\rho^*} + 2k^{*4} U^{*2}\upsilon^*\end{array}\right] + \left[\begin{array}{l}\left\{2k^{*3}U^*\nu_0^* - k^{*2}U^{*2}\right\} \\ \left\{f_s^{*2} + k^{*2}U^{*2}\right\} - \\ f_s^{*2} k^{*2} U^{*2}\alpha_0 + \\ \dfrac{T^* k^{*3}}{2\rho^*}\left(f_s^{*2} + k^{*2}U^{*2}\right)\end{array}\right] = 0 \quad (18)$$

The graphical analysis between the growth rate (n*) and wave number (k*) is done to study the effects of surface tension, suspended particle along with FLR correction on the Kelvin Helmholtz instability of two superposed incompressible viscous fluids.

Graph (1) is plotted between growth rate (n*) and wave number (k^*) at different values of surface tension (T^*) here we have put $f^* = 0.4$, $\alpha_0^* = 0.2$, $\nu^* = 0.8$, $\nu_0^* = 0.2$, $U^* = 0.6$, $\rho^* = 0.6$ and $T^* = 0.2, 0.4, 0.5, 0.6$ in dimensionless equation (18) and we found from the figure (1) that the growth rate is decreasing on raising the value of surface tension (T^*) or we can say that surface tension shows stabilizing effect on the growth rate of K-H instability.

From figure (2) we noticed the effect of suspended particles on the K-H instability and here we plotted the graph by putting $f^* = 0.4$, $\nu^* = 0.8$, $\nu_0^* = 0.2$, $U^* = 0.7$, $T^* = 0.2$, $\rho^* = 0.6$ and $\alpha_0^* = 0.1, 0.2, 0.3, 0.4$ in the dimensionless equation and found that the growth rate is increasing on raising the value of suspended particle density.

We obtained the graph (3) by putting $\nu^* = 0.8$, $\alpha_0^* = 0.2$, $\nu_0^* = 0.2$, $U^* = 0.6$, $\rho^* = 0.6$, $T^* = 0.2$ and $f^* = 0.2, 0.3, 0.4, 0.5$ in dimensionless equation and found that the growth rate is increasing on raising the value of f*.This implies that the suspended particle has destabilizing effect on the growth rate of KH instability.

V. Conclusion

In the present investigation, Kelvin Helmholtz instability of two incompressible fluids in porous medium has stabilizing influences in the variation of surface tension. But suspended particle has destabilizing influence.

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