

Image Denoising Based on SURE Using OWT

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Abstract - An overview of orthonormal wavelet image denoising is discussed here. Instead of postulating a statistical model for the wavelet coefficients, denoising process is directly parameterized as a sum of elementary nonlinear process with unknown weights. Then an estimate the mean square error between original image and denoising image is minimized. A statically unbiased MSE estimate stein's unbiased risk estimates (SURE) that depends on the noisy image alone is considered. This estimate is quadratic in unknown weights and its minimization amounts to solving a linear system of equations. The existence of this a priori estimate makes it unnecessary to device a specific statistical model for the wavelet coefficient. This estimate turns out to be more accurate as more data available which is the cost of images. Here an interscale orthonormal wavelet thresholding algorithm based on the new SURE approach is implemented. Test result shown improved PSNR compared to existing state of art procedures.

Keywords: OWT, DWT, Thresholding, PSNR, SURE.

I. Introduction

An image may be defined as a two-dimensional function $f(x, y)$, where x & y are spatial coordinates, & the amplitude of f at any pair of coordinates (x, y) is called the intensity or gray level of the image at that point. When x, y & the amplitude values of f are all finite discrete quantities, we call the image a digital image. The field of DIP refers to processing digital image by means of digital computer. Digital image is composed of a finite number of elements, each of which has a particular location & value. The elements are called pixels [1]. Vision is the most advanced of our sensor, so it is not surprising that image play the single most important role in human perception. However, unlike humans, who are limited to the visual band of the EM spectrum imaging machines cover almost the entire EM spectrum, ranging from gamma to radio waves. They can operate also on images generated by

sources that humans are not accustomed to associating with image.

II. Image Denoising

II.1. Denoising

De-noising plays a vital role in the field of the image pre-processing. It is often a necessary to be taken, before the image data is analyzed. The main aim of an image-denoising algorithm is then to reduce the noise level, while preserving the image features. The multi resolution analysis performed by the wavelet transform has been shown to be a powerful tool for denoising. In wavelet domain, the noise is uniformly spread throughout the coefficients, while most of the image information is concentrated in the few largest coefficients. The most straight forward way of distinguishing information from noise in the wavelet domain consists of thresholding the wavelet coefficients [4].

II.2. Wavelet

A Wavelet is a waveform of efficiently limited duration that has an average value zero. Compare wavelets with sine wave, which are the basis of Fourier analysis. Sine waves so not have limited duration, wavelets tend to be irregular and asymmetric [5].

II.3. Thresholding

The thresholding is classified into two types. They are:

II.3.1.Hard thresholding

Hard thresholding can be defined as,

$$D(U, \lambda) = \begin{cases} U & \text{for all } |U| > \lambda \\ 0 & \text{otherwise} \end{cases} \quad (1.1)$$

Hard threshold is a "keep or kill" procedure and is more intuitively appealing. The transfer function of the hard thresholding is shown in the figure. Hard thresholding may seem to be natural. Hard thresholding does not even work with some algorithm such as GCV procedure. Sometimes pure noise coefficients may pass the hard threshold and appear as annoying 'blips' in the output.

II.3.2.Soft thresholding

Soft thresholding can be defined as follow,

$$D(U, \lambda) = \text{sgn}(U) \max(0, |U| - \lambda) \quad (1.2)$$

Soft threshold shrinks coefficients above the threshold in absolute value. The false structures in hard thresholding can overcome by soft thresholding. Now a days, wavelet based denoising methods have

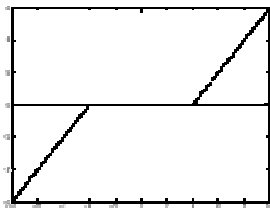


Fig.1.

received a greater attention. Important features are characterized by large wavelet coefficient across scales, while most of the timer scales.

III. Thresholding Technique

III.1 VisuShrink

Visushrink is thresholding by applying the universal threshold proposed by Donoho and Johnstone. This threshold is given by

$$T = \sqrt{(\sigma^2 \log M)} \quad (1.3)$$

where σ is the noise variance and M is the number of pixels in the image. For denoising images, visushrink is found to yield an overly smoothed estimated. It is because of Universal threshold (UT) is derived under the constraint that with high probability, the estimate should be at least as smooth as the signal. So Universal threshold (UT) tends to be high for large values of M , killing many signal coefficients along with the noise. Thus the Threshold does not adapt well to discontinuities in the signal [7].

III.2 Sure Shrink

Sure shrink is a thresholding by applying sub band adaptive threshold, a separate threshold is computed for each detail sub band based upon SURE (Stein's Unbiased Risk Estimate), a method for estimating the

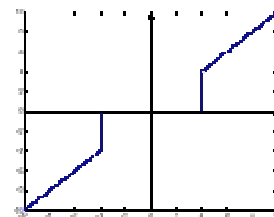


Fig.2.

loss $\|\hat{\mu} - \mu\|^2$ in an unbiased fashion. Let the wavelet coefficients in the j th sub Band be $\{X_i : i = 1, 2, \dots, d\}$, $\hat{\mu}$ is the soft threshold estimator $\hat{X}_i = \eta_i(X_i)$.

By applying stein's unbiased estimate of the risk

$$E \|\hat{\mu}^{(s)}(x) - \mu\|^2$$

$$SURE(t; X) = d - 2\#\{i: |X_i| \leq t\} + \sum_{n=1}^d \min(|X_n|, t)^2 \quad (1.4)$$

Here threshold t^s that minimizes SURE (t: x)

$$t^S = \operatorname{argmin} SURE(t : X) \quad (1.5)$$

The result is much better than visushrink. The sharp features of image are retained. But MSE is considerably lower. This is because sureshrink is sub band adaptive [6].

III.3. BayesShrink

Bayes shrink is an adaptive data driven threshold for image denoising via wavelet soft thresholding. The threshold is driven in a Bayesian frame work and its assume Generalized Gaussian distribution (GGD) for the wavelet coefficient in each detail sub band and try to find the threshold T minimizes the Bayesian Risk. The BayesShrink performs better than sure shrink in terms of MSE [5][8].

The above methods give best results on soft thresholding Comparison of denoising MSE for sure shrink, BayesShrink, Visushrink for hard and soft thresholding was shown in figure 2.3.

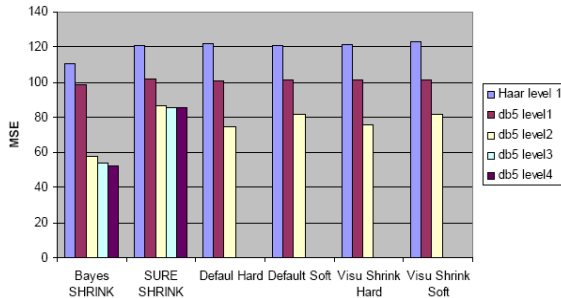


Fig.3. Image Denoising Methods Vs MSE

Beyond the Point wise approach, more recent investigations have shown that substantially larger denoising gains can be obtained by considering the Intra and Inter scale correlations of the wavelet coefficients.

IV. SURE-BASED THRESHOLDING

Here construction of an efficient parameterized a wavelet denoising function $\theta(y)$ is proposed. A nonlinear transition is required between low-

magnitude and high-magnitude co-efficient are required. The difficulty is to choose suitable basic function Φ_k that will determine the shape of the denoising function. The denoising function $\theta(y)$ to satisfy the following properties Differentiability, Anti-symmetry, Linear behavior for a large coefficient. To satisfy above conditions, a point wise denoising function is to be chosen $\theta(y)$ it is Derivatives of Gaussians (DOG) because they decay quite fast which shows a linear behavior close to the identity for large coefficients.

$$\theta(y) = \sum_{k=1}^K a_k y \ell^{-\frac{(k-1)y^2}{2T^2}} \quad (1.6)$$

Denoising function contain two non linear dependencies are the number of terms K and parameter T. If K=1 one parameter, the denoising function simply becomes $\theta(y) = a_1 y$, which is the simplest linear point wise denoising function. The direct minimization of the estimate ϵ provides [12]

$$a_1 = 1 - \frac{\sigma^2}{\langle y^2 \rangle} \quad (1.7)$$

This is known as the James–Stein estimator. Practical tests on K=1 it leads to non linear relationship between noisy data and denoised data. The optimal value of the parameter T is closely linked to the standard deviation σ of the noise and in a lesser way to the number of parameters K .The denoising function is much more flexible than the soft threshold. The both number of terms K and the parameter T have only a minor influence on the quality of the denoising process. So K and T can be fixed once for all, independently of the type of image. From practical point of view, K=2 terms and T=

$\sqrt{(6\sigma)}$. Point wise thresholding function leads to

$$\theta_0(y; a) = \left(a_1 + a_2 \ell^{-\frac{y^2}{12\sigma^2}} \right) y \quad (1.8)$$

V. SURE ALGORITHM

Wavelet denoising consists of three main stages are,

1. Perform DWT to the noisy data $y=(y_n)_{n \in [1,N]}$ which is the sum of the noise and free data $x=(x_n)_{n \in [1,N]}$ and the noise $b=(b_n)_{n \in [1,N]}$

2. Denoise J noisy wavelet sub images $y_n^j = x_n^j + b_n^j$. $(y_n^j)_{n \in [1, N]}$ by computing J estimate \hat{x} of the noise free high pass sub bands x_n^j .
3. Reconstruct the denoised image by applying the inverse discrete wavelet transform (IDWT) on the processed high pass wavelet sub images to obtain an estimate of the noise-free data [9].

V.1. Steins unbiased MSE Estimate (SURE)

The performance of denoising is measured in terms of Peak signal-to noise ratio (PSNR).

$$PSNR = 10 \log_{10} \left(\frac{\max(x^2)}{\langle |\hat{x} - x|^2 \rangle} \right) \quad (1.10)$$

Since noise is random process, expectation is introduced $\langle \cdot \rangle$ to guess the potential results obtained after processing the noisy data y . The aim of image denoising is naturally to maximize the PSNR and to minimize the MSE. Estimate of each sub band \hat{X}_i

by a Point wise function of y_i .

$$\hat{(x_n^j)}_{n \in [1, N_j]} = (\theta^j(y_n^j))_{n \in [1, N_j]} \quad (1.11)$$

To check whether function θ that minimizes the error is given by MSE. The only parametric term to estimate in the above equation is $\langle x\theta(y) \rangle$ other parametric term is independent of noise. To estimate $\langle x\theta(y) \rangle$ steins proposed a new theorem it was discussed below [9].

VI. EXPERIMENTAL RESULT

The various images are used for denoising which are representative set of standard 8-bit grayscale images such as Boat, Einstein, Lena, All corrupted by simulated additive Gaussian white noise at eight different power levels $\sigma \in [5, 10, 15, 20, 25]$ which corresponds to PSNR decibel values [34.15, 28.13, 24.61, 22.11, 20.17]. The denoising process has been performed over five different noise realizations for each standard deviation and the resulting PSNRs

averaged over these five runs. The Table I shows the PSNR values.

TABLE I

IMAGE	σ	I/P SNR	OUTPUT SNR			
			VISUSHRINK	BAYESHSHRINK	SURE	PROPOSED METHOD
LENA	20	22.11	31.28	31.20	31.30	31.37
	25	20.17	30.03	30.58	31.46	31.58
BOAT	20	22.11	28.11	28.38	29.00	29.47
	25	20.17	28.05	28.43	29.30	29.46
EINSTIEN	20	22.11	28.40	28.93	30.16	32.09
	25	20.17	27.20	27.72	28.94	29.00

VII. Conclusion

Performance of denoising algorithms is measured using quantitative performance measures such as peak signal-to-noise ratio (PSNR), signal-to-noise ratio (SNR) as well as in terms of visual quality of the images. We then investigated many soft thresholding schemes such as Visushrink, Sureshrink and BayesShrink for denoising images. We found that sub band adaptive thresholding performs better than a universal thresholding. An important point to note is that although Sureshrink performed better than BayesShrink, it adapts well to sharp discontinuities in the signal. Among these, SURE shrink gave the best results.

The efficiency of new proposed SURE-based approach, which gave the best, output PSNRs for most of the images.

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